Nonmonotonic Inheritance in Object-Oriented Deductive Database Languages

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Deductive Databases + **(Nonmonotonic) Inheritance**

Object orientation:	Default Logic, Extension semantics,
F-Logic, Horn programs,	special type of defaults
(forward-chaining evaluation)	

deductive object-oriented database languages

- deduction can take place depending on inherited facts
  $\Rightarrow$ indirect conflicts.
- class hierarchy and -membership is subject to deduction.

F-Logic in a Nutshell

Atoms:

Object isa Class	tweety isa penguin	% ISA-relation, "$\in$"
SubClass :: Class	penguin :: bird	% SUBCLASS-relation "$\subseteq$"
Object[Method@(Params)→R]		% single-valued object method
tweety[lives→"Antarctica"]		
Object[Method@(Params)•→R]		% single-valued inheritable method
bird[fly •→true; laying_eggs •→true]	penguin[fly •→false]	

- Variables can occur at arbitrary positions.
- Formulas, rules, and programs defined as usual.
- Implemented: T_P-based bottom-up evaluation.
- Semantics: H-structures; similar to Herbrand structures (+ subclass transitivity)
  Consistency: not o[m→v] and o[m→w] for v $\neq$ w.

Inheritance

$$\dfrac{\dfrac{\begin{array}{l}\text{bird[laying\_eggs} \bullet\!\!\to \text{true]} \\ \text{penguin} :: \text{bird}\end{array}}{\text{penguin[laying\_eggs} \bullet\!\!\to \text{true]}} \\ \text{tweety isa penguin}}{\text{tweety[laying\_eggs} \to \text{true]}}$$

$$\dfrac{\dfrac{\begin{array}{l}\text{bird[fly} \bullet\!\!\to \text{true]} \\ \text{penguin} :: \text{bird} \\ \text{penguin [fly} \bullet\!\!\to \text{false]}\end{array}}{\text{penguin[fly} \bullet\!\!\to \text{false]}} \\ \text{tweety isa penguin}}{\text{tweety[fly} \to \text{false]}}$$

Indirect Conflict

r_nixon isa republican, republican[policy•→hawk],
mrs_nixon[policy→pacifist], mrs_nixon[husband→r_nixon],
W[policy→P] ← W[husband→O], O[policy→P].

republican[policy•→hawk].
r_nixon isa republican.

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r_nixon[policy→hawk].
mrs_nixon[husband→r_nixon].
W[policy→P] ← W[husband→O], O[policy→P].

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mrs_nixon[policy→hawk].
mrs_nixon[policy→pacifist].
inconsistent !!

⇝ policy of r_nixon must remain undefined.

Default Logic

Defaults:

$$d = \frac{\alpha : \beta_1, \ldots, \beta_n}{w}$$

- *precondition* $p(d) = \alpha$,
- *justification* $J(d) = \beta = \{\beta_1, \ldots, \beta_n\}$,
- *consequence* $c(d) = w$.

Given α, if β can be assumed consistently, one can conclude w.

Inheritance as Defaults:

$$\frac{O \text{ isa } C,\ C[M \bullet\!\!\rightarrow V] \ :\ \neg\exists C'(O \text{ isa } C' \wedge C' :: C)\ ,\ O[M \rightarrow V]}{O[M \rightarrow V]}$$

$$\frac{SC :: C,\ C[M \bullet\!\!\rightarrow V] \ :\ \neg\exists C'(SC :: C' \wedge C' :: C)\ ,\ SC[M \bullet\!\!\rightarrow V]}{SC[M \bullet\!\!\rightarrow V]}$$

Extensions

Let $\Delta = (D, F)$ be a default theory.

Then, for all sequences $S_0 = F, S_1, S_2, \ldots, S_n$ of sets of formulas s.t. $S = (\bigcup_{i=0}^{\infty} S_i)$ and

$$S_{i+1} = S_i \cup c(GD(S\ , S_i, D))\ ,$$
$$AD_{i+1} = AD_i \cup GD(S\ , S_i, D)\ ,$$

where $GD(S, S_i, D) :=$

$$:= \{d \text{ such that } d \text{ is an instance of a default in } D,$$
$$\mathrm{Th}(S_i) \models p(d)\ , \text{ and}$$
$$\mathrm{Th}(S \cup \{\beta\}) \text{ is consistent for every } \beta \in J(d)\}\ ,$$

$\mathrm{Th}(S)$ is an extension of Δ.

"*quasi-inductive* definition" (S used in Definition of S_{i+1})

- $S = F \cup \bigcup_{i=0}^{\infty} c(AD_i)$,
- no default applicable in $\mathrm{Th}(S)$.

Inflationary Extensions

Let $\Delta = (D, F)$ be a default theory.

Then, for all sequences $S_0 = F, S_1, S_2, \ldots, S_n$ of sets of formulas s.t. $S = (\bigcup_{i=0}^{\infty} S_i)$ and

$$S_{i+1} = S_i \cup c(d) \ ,$$
$$AD_{i+1} = AD_i \cup d \ ,$$

where $d \in GD(—, S_i, D) :=$

$:= \{d$ such that d is an instance of a default in D,

$\text{Th}(S_i) \models p(d)$, and

$\text{Th}(S_i \ \cup \{\beta\})$ is consistent for every $\beta \in J(d)\}$,

$\text{Th}(S)$ is an inflationary extension of Δ.

replace "$\text{Th}(S \cup \{\beta\})$ is consistent" with "$\text{Th}(S_i \cup \{\beta\})$ is consistent"; e.g. [Marek/Truszczynski 93].

- $S = F \cup \bigcup_{i=0}^{\infty} c(AD_i)$,
- no default applicable in $\text{Th}(S)$.

Complete, but not correct:

Proposition 1 (Extensions vs. Inflationary Extensions)
Let $\Delta = (D, F)$ be a Default theory.

1. *Every extension S of Δ is also an inflationary extension of Δ, and*
2. *Let S be an inflationary extension. If for every $\beta \in J(AD_n)$, β is consistent with S, then S is an extension of Δ.*

Inflationary: a default which has been once applied is not undone if in a later step one of its *justifications* turns out to be wrong (above criterion).

- forbidding the application of defaults whose justifications will be falsified later,
- forbidding the application of a default whose logical consequences would falsify the justifications of another default which has been applied earlier.

Cautious Inflationary Extensions

Let $\Delta = (D, F)$ be a default theory.

Then, for all sequences $S_0 = F, S_1, S_2, \ldots, S_n$ of sets of formulas s.t. $S = (\bigcup_{i=0}^{\infty} S_i)$ and

$$S_{i+1} = S_i \cup c(d) \ ,$$
$$AD_{i+1} = AD_i \cup d \ ,$$

where $d \in GD(AD_i, S_i, D) :=$

$:= \{d$ such that d is an instance of a default in D,
$\mathrm{Th}(S_i) \models p(d)$, and
$\mathrm{Th}(S_i \cup \{\beta\})$ is consistent for every $\beta \in J(d)$,
and $\mathrm{Th}(S_i \cup c(d) \cup \beta)$ is consistent
for every $\beta \in J(AD_i)\}$.

$\mathrm{Th}(S)$ is a cautious inflationary extension of Δ.

- $S = F \cup \bigcup_{i=0}^{\infty} c(AD_i)$,
- possibly applicable defaults in $\mathrm{Th}(S)$.

Proposition 2 (Cautious Inflationary vs. Inflationary Extensions)
Let $\Delta = (D, F)$ be a default theory.

- *Every cautious inflationary extension S of Δ can be extended to an inflationary extension.*
  If $GD(S, D) = \emptyset$, then S is an inflationary extension.
- *If an inflationary extension S satisfies Prop. 1(2) then S is also a cautious inflationary extension.*

Proof: Prefixes of computation sequences.

Proposition 3 (Extensions vs. Cautious Inflationary Extensions)
Given a default theory $\Delta = (D, F)$, a cautious inflationary extension S of Δ is an extension of Δ if $GD(S, D) = \emptyset$.

The Horn Case

- $\Delta = (D, P)$,
- P Horn formulas,
- D Inheritance Defaults, i.e. of the form

$$\frac{O \text{ isa } C, C[M \bullet\!\!\rightarrow V] \ : \ \neg\exists C'(O \text{ isa } C' \wedge C' :: C) \ , \ O[M \rightarrow V]}{O[M \rightarrow V]}$$

$\Rightarrow$ the "extension base" $S = P \cup \bigcup_{i=0}^{\infty} c(AD_i)$ is Horn.

Definition 1 Given an F-Logic program P and an extension base S of Δ_P,

$$\mathcal{H} := T_S^{\omega}(\emptyset)$$

is called the *H-extension* of P to S (analogous for *inflationary H-extensions* and *cautious inflationary H-extensions*).

Computing Inflationary H-extensions

Proposition 4 *Let P be an F-Logic program, Δ_P its default theory.*

Then, for all sequences $\mathcal{H}_0 = T_P^\omega(\emptyset), \mathcal{H}_1, \mathcal{H}_2, \ldots$ of sets of H-structures s.t. $\mathcal{H} = (\bigcup_{i=0}^{\infty} \mathcal{H}_i)$ and

$$\mathcal{H}_{i+1} = T_P^\omega(\mathcal{H}_i \cup c(d)) \ ,$$
$$AD_{i+1} = AD_i \cup d$$

where $d \in GD(\mathcal{H}_i, \Delta_P) :=$

$:= \{d$ *such that d is a ground instance of a default in D_P,*

$p(d) \subseteq \mathcal{H}_i$ *, and*

$Th(F \cup \mathcal{H}_i \cup \{\beta\})$ *is consistent for every* $\beta \in J(d)\}$,

$\mathcal{H}$ is an inflationary H-extension.

Proposition 5 *Let $\mathcal{H}$ be an inflationary H-extension computed by the above algorithm.*
If for every $\beta \in J(AD_n)$, β is consistent with $\mathcal{H}$, then $\mathcal{H}$ is an H-extension of Δ.

Inheritance Triggers

alternatingly computing a classical deductive fixpoint and carrying out a specified amount of inheritance.

Definition 2 (Inheritance Triggers) $\mathcal{H}$ an H-structure.

- An *inheritance trigger* in $\mathcal{H}$: $(o \; \mathsf{isa} \; c, m {\bullet\!\!\to} v)$ s.t.
  - $(o \; \mathsf{isa} \; c) \in \mathcal{H}$,
  - and $c[m {\bullet\!\!\to} v] \in \mathcal{H}$,
  - no $o \neq c' \neq c$ s.t. $\{o \; \mathsf{isa} \; c', c' :: c\} \subseteq \mathcal{H}$.
  - (analogous for $::$).
- An inheritance trigger $(o \; \mathsf{isa} \; c, m {\bullet\!\!\to} v)$ is *active*: no v' s.t. $o[m {\to} v'] \in \mathcal{H}$.
- $\mathbf{T}(\mathcal{H})$: set of active inheritance triggers in $\mathcal{H}$.
- Application ("firing") of a trigger:

$$\left.\begin{array}{l} t = (o \; \mathsf{isa} \; c, m {\bullet\!\!\to} v) \\ t = (c' :: c, m {\bullet\!\!\to} v) \end{array}\right\} \leadsto t(\mathcal{H}) := \left\{\begin{array}{l} \mathcal{H} \cup \{o[m {\to} v]\} \\ \mathcal{H} \cup \{c'[m {\bullet\!\!\to} v]\} \end{array}\right.$$

- $\mathcal{I}_P^t(\mathcal{H}) := T_P^\omega(t(\mathcal{H}))$
  one step inheritance transformation.

Definition 3 (Inheritance-Canonic Model)
Let P be an F-Logic program P.

A sequence $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_n$ of H-structures is an $\mathcal{I}_P$*-sequence* if

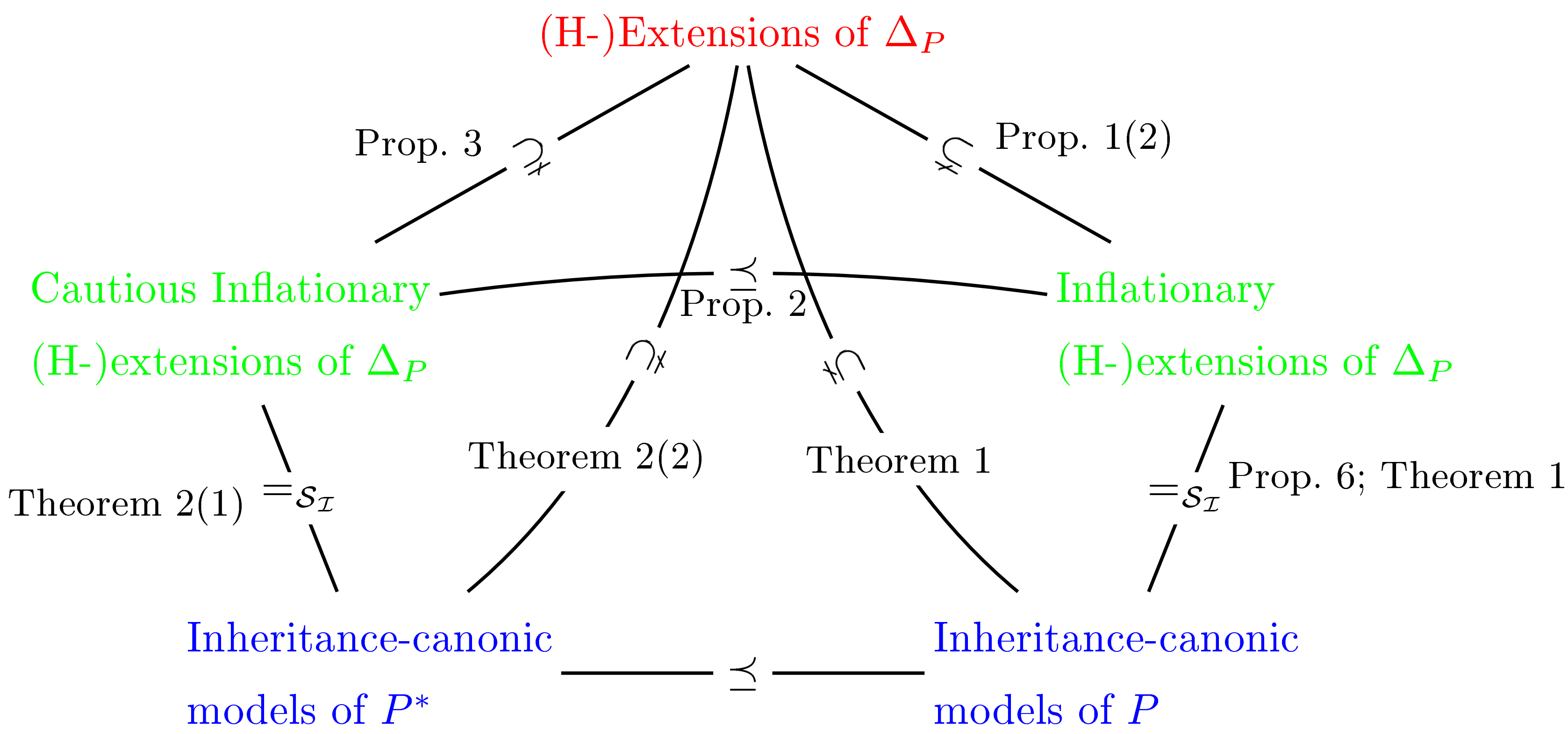
- $\mathcal{M}_0 = T_P^\omega(\emptyset)$ and
- for all i, there is a $t_i \in \mathbf{T}(\mathcal{M}_i)$ s.t. $\mathcal{M}_{i+1} = \mathcal{I}_P^{t_i}(\mathcal{M}_i)$.

$\mathcal{M}$ is an *inheritance-canonic* model of P if there is an $\mathcal{I}_P$-sequence $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M} \neq \bot$ s.t. $\mathcal{M}$ has no active triggers.

The consistency check before inheriting is omitted in the definition of inheritance-canonic models:

Definition 4 Let $\mathcal{S}_\mathcal{I}(P)$ be the set of H-structures $\mathcal{H}$ s.t. there exists an $\mathcal{I}_P$-sequence $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{H}$, and $\mathcal{I}_P^t(\mathcal{H}) = \bot$ for every $t \in \mathbf{T}(\mathcal{H})$.

Comparison



$M_1 \preceq M_2$: every structure/theory in M_1 can be extended to one in M_2.

Computation of inheritance-canonic models implements the computation of inflationary H-extensions:

Proposition 6 *P an F-Logic program, Δ_P the corresponding default theory. Then the following sets coincide:*

- *the set of $\mathcal{I}_P$-sequences $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_n$ s.t. $\mathcal{M}_n \neq \bot$, and*
- *the set of prefixes $\mathcal{H}_0, \mathcal{H}_1, \ldots, \mathcal{H}_n$ of computations of inflationary H-extensions.*

Theorem 1
($\mathcal{I}_P$-sequences and inflationary H-Extensions)

- *$\mathcal{S}_{\mathcal{I}}(P)$ is the set of inflationary H-extensions of P.*
- *An H-structure $\mathcal{H} \in \mathcal{S}_{\mathcal{I}}(P)$ is an H-extension of P if and only if there is an $\mathcal{I}_P$-sequence $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{H}$ which satisfies Prop. 5.*

Cautious Strategy

D_{inh}: justification can only be annulled in later steps is when an intermediate class is inserted:

$$\frac{O \text{ isa } C,\ C[M \bullet\!\!\to V] \ :\ \neg\exists C'(O \text{ isa } C' \wedge C' :: C)\ ,\ O[M \to V]}{O[M \to V]}$$

Proposition 7 (Static Class Hierarchy) *Let P be an F-Logic program P with a static class hierarchy. Then, the set of extensions of Δ_P and the set of inflationary extensions of Δ_P coincide.*

Enforce cautious computations: with every instance of inheritance, the class hierarchy at this point is fixed by forbidding the introduction of an intermediate class.

$$\frac{O \text{ isa } C, C[M \bullet\!\!\to V] \ :\ \neg\exists C'(O \text{ isa } C' \wedge C' :: C)\ ,\ O[M \to V]}{\neg\exists C'(O \text{ isa } C' \wedge C' :: C)\ ,\ O[M \to V]}$$

Proposition 8 *For an F-Logic program P, every inflationary extension of Δ^*_P is also an extension of Δ^*_P.*

Cautious Trigger Mechanism

Adding a rule

$$r(t) \;:=\; \mathsf{inconsistent} \leftarrow \mathsf{o}\,\sharp\,\mathsf{C},\ \mathsf{C} :: \mathsf{c},\ \mathsf{not}\ (\mathsf{c}{=}\mathsf{C}).$$

as an *integrity constraint* to the program whenever an inheritance trigger $t = (o\sharp c, m{\bullet}\!\!\rightarrow\! v)$ is fired.

Definition 5 P an F-Logic program P. A sequence $\mathcal{M}_0, \mathcal{M}_1, \ldots, \mathcal{M}_n$ of H-structures is an ${\mathcal{I}^*}_P$*-sequence* if

- $\mathcal{M}_0 = T_P^\omega(\emptyset)$ and
- for all i, there is a $t_i \in \mathbf{T}(\mathcal{M}_i)$ s.t. $\mathcal{M}_{i+1} = \mathcal{I}^{t_i}_{P_{i+1}}(\mathcal{M}_i) \neq \bot$ where $P_0 = P$ and $P_{i+1} = P_i \cup r(t_i)$.

Theorem 2 ($\mathcal{I}^*_P$-sequences and cautious H-Extensions) *Let P be an F-Logic program. Then,*

1. *$\mathcal{S}^*_{\mathcal{I}}(P)$ is the set of cautious inflationary H-extensions of P.*
2. *for every $\mathcal{H}$ in $\mathcal{S}^*_{\mathcal{I}}(P)$, if $\mathcal{I}^t_P(\mathcal{H}) = \bot$ for every $t \in \mathbf{T}(\mathcal{H})$, then $\mathcal{H}$ is an H-extension of P (equiv. to $GD(\mathcal{H}, \Delta_P) = \emptyset$).*

Conclusion

- integrating nonmonotonic inheritance into a deductive object-oriented database language.
- T_P-based computation of Herbrand-like structures which approximate the extensions of the default theory corresponding to P.
- investigation of the current F-Logic definition/implementation for inheritance: approximately correct:
- correct in all cases where the specification is "well-behaved"; post-computation check by Theorem 2.