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Theory I - Exercise sheet 3

Submission due to June 3, 2008 before the tutorial starts.
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Exercise 1: $(2 + 2$ points) Hashing: chaining

a) Insert the key sequence $7, 18, 2, 3, 14, 25, 1, 11, 12, 1332$ with hashing by chaining in a hash table with size 11. Please show the final table by using the hash function $h(k) = k \ mod\ 11$.

b) Consider a hash table that is an array of length 31 with overflow lists (for chaining) and the hash function

$$h(n) = \lfloor n \cdot \log_2 n \rfloor \mod 31$$

Is this a good hash function? Assume that the keys to be inserted are integers between 1 and 1000 and that they are equally distributed.
Hint: You may want to write a short program in order to solve this exercise.

Exercise 2: $(2 + 2 + 2 + 2$ points) Hashing: open addressing
Give the allocation for a hash table of size 17, if sequentially the keys

$$2, 32, 43, 16, 77, 51, 1, 17, 42, 111$$

were inserted with hash function $h(k) = \ k \ mod\ 17$ used by

(i) linear probing

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

(ii) quadratic probing

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

(iii) double hashing with $h'(k) = 1 + k \; mod \; 13$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

(iv) double hashing with Brent's algorithm ($h'(k) = 1 + k \; mod \; 13$)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Exercise 3: ($1 + 2 + 2$ points) Universal Hashing

We consider universal hashing for the universe $U = \{0, \cdots, 10\}$ of size $N = 11$. For a hash table of size $m = 4$ we draw randomly the hash function

$$h_{a,b}(x) = ((ax + b) \; mod \; N) \; mod \; m$$

with $a = 8$ and $b = 3$.

a) Please describe in your own words the idea of universal hashing.

b) Please give for the key sequence $S = \{1, 5, 8, 9\}$ the allocation of the hash table. Use hashing by chaining.

c) Find the "worst" hash function $h_{a,b}$ for S, meaning the values a and b, so that by hashing with $h_{a,b}$ at least 3 elements of the key sequence $S = \{1, 5, 8, 9\}$ will be mapped to the same place in the hash table.

Exercise 4: (3 points) Fibonacci numbers

Consider the function $S : \mathbf{N} \to \mathbf{N}$, defined as follows:

$$S(n) = \begin{cases} 1 & \text{if } n = 0 \\ 2 + \sum_{i=2}^{n} S(i-2) & \text{if } n \geq 1 \end{cases}$$

(If $n < 2$ in the summation on the right, the sum is empty and hence, equals 0.)

Show that $S(n) = F_{n+2}$ for all $n \geq 0$, where F_k is the k-th Fibonacci number.