$$F = \{ A \to B, \boxed{B \to C} \} \qquad U = \{A\,B\,C\}$$

$A,$

$V = \{ABC\}$

$F = \{A \to B,\ B \to C,\ \boxed{C \to A}\}$

$V_1 = AB \qquad V_2 = \underline{BC}$

$\pi_{\{AB\}} F$

$= A \to B,\ \underline{B \to A}$

$\pi_{\{BC\}} F$

$= B \to C,\ C \to B$

$\left\{ \begin{array}{l} A \to B,\ B \to A \\ B \to C,\ C \to B \end{array} \right\} \equiv F$

$\underset{\pi}{}$

$\underline{C \to A}$

$\{A\ B\ C\}$

$F = \{AB \to C$

$\underline{C \to A}\}$

Key: AB, $\overbrace{CB}$?

$\underline{(CA)}$ (CB)

Key: C : $\pi_{CA} F = \{C \to A\}$

Key: BC $\pi_{CB} F = \phi$ $B \to C$ X
$C \to B$ X

$V = \{A\ B\ C\}$

$F = \{A \to B,\ B \to C,\ A \to C\}$

$AB,\ BC,\ CA$

$\{A \to B,\ B \to C$

$F = \{A \to \cancel{B}\cancel{C},\ B \to C\}$

$A \to B,\ A \to C,\ B \to C$

$F' \equiv F$

$$F = \{ \underbrace{A \to B,\ B \to C}_{\Delta},\ \cancel{\boxed{A \to C}} \}$$

$$F = \Delta \cup \{A \to C\}$$

$$F' = \Delta$$

$$F \equiv F'$$

$$= F' \models A \to C$$

$F = \{ AB \to C$

$A \to B \}$

F_1: $A \to B$, $\cancel{C \to B}$

$\equiv$

F_2: $AC \to B$

(1) $F_1 \models F_2$ (2) $F_2 \models F_1$

$A \to B \models AC \to B$

$AC \to BC$

$\models AC \to B$, $\boxed{\cancel{AC \to C}}$

$V = \{ A B C \}$

$\mathcal{F} = \{ \underline{A}\,\underline{B} \to C, \; A \to B \}$

$\mathcal{F}' = \{ A \to C, \; A \to B \}$

$$\underline{\mathcal{F} \models \mathcal{F}'} \equiv \underline{\mathcal{F} \models A \to C}$$

$$\underline{\mathcal{F}' \models \mathcal{F}}$$ always hold

$$\frac{AB \to C}{A \to B} \models A \to C$$

$$\frac{AA \to AB}{A \to AB}$$

$$xplus(X, Y, F')$$

$\Rightarrow$ right: $F' \models X \rightarrow Y \setminus Z$

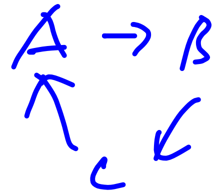
left: $F \models Z \rightarrow Y$

$$\underline{AB \rightarrow CD} \quad Y$$

$$\underline{AB \rightarrow C} \quad Z$$

$\{\ A \to B,\ \underline{B \to A},\ B \to C$

$\underline{\cancel{A \to C}},\ C \to A\ \}$

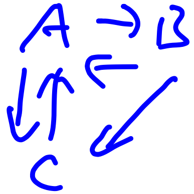


$\{A \to B,\ B \to C\} \models A \to C$

$\{B \to C,\ C \to A\} \models B \to A$

$\{\ A \to B,\ B \to A,\ \underline{\cancel{B \to C}}$

$\underline{A \to C},\ \underline{C \to A}\ \}$



$\underbrace{\{A \to B,\ B \to A,\ C \to A\}}_{F'} \models \cancel{A \to C}$

$A \to BC$

$\underline{\text{xplus}(A,\ BC,\ F')}$